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ALGEBRA.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

66. Proposed by A. H. BELL, Box 184, Hillsboro, Illinois.

Solve the equations:

$$a^2x=(2x^2-a^2)\sqrt{x^2+y^2} \dotfill (1),$$

$$b^2y=(2y^2-b^2)\sqrt{x^2+y^2} \dotfill (2).$$

I. Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas-Texas.

Let $x=r\cos\theta$, $y=r\sin\theta$. Then the equations become

$$a^2\cos\theta=2r^2\cos^2\theta-a^2 \dotfill (1),$$

$$b^2\sin\theta=2r^2\sin^2\theta-b^2 \dotfill (2).$$

Eliminating r^2 from (1) and (2) we get

$$\frac{b^2(1+\sin\theta)}{1+\cos\theta}=a^2\tan^2\theta \dotfill (3).$$

Now $\sin\theta=\dfrac{2\tan\frac{1}{2}\theta}{1+\tan^2\frac{1}{2}\theta}$, $\cos\theta=\dfrac{1-\tan^2\frac{1}{2}\theta}{1+\tan^2\frac{1}{2}\theta}$. $\therefore\ b^2(1+\tan\frac{1}{2}\theta)^2=2a^2\tan^2\theta$,

$$\text{or } b(1+\tan\tfrac{1}{2}\theta)=\pm\sqrt{2}\, a\tan\theta \dotfill (4).$$

Let $z=\tan\frac{1}{2}\theta$; then (4) becomes

$$z^3+z^2-\left(1\mp\frac{2\sqrt{2a}}{b}\right)z=1 \dotfill (5).$$

Let $u=z-\frac{1}{3}$; then (5) becomes

$$u^3-\tfrac{2}{3}\left(2\mp\frac{3\sqrt{2a}}{b}\right)u=\tfrac{2}{27}\left(8\pm\frac{9\sqrt{2a}}{b}\right) \dotfill (6).$$

When a and b are known we can find u from (6), after which z and r and finally x and y become known.

II. Solution by HENRY HEATON, M. Sc., Atlantic, Iowa.

As in preceding solution,

$$a^2\cos\theta=2r^2\cos^2\theta-a^2 \dotfill (3),$$

$$b^2\sin\theta=2r^2\sin^2\theta-b^2 \dotfill (4).$$

$$\therefore\ 2r^2\cos^2\theta = a^2(1+\cos\theta) \ldots\ldots(5), \text{ and } 2r^2\sin^2\theta = b^2(1+\sin\theta) \ldots\ldots(6).$$

Dividing (5) by (4), $\tan^2\theta = \frac{b^2}{a^2}\left(\frac{1+\sin\theta}{1+\cos\theta}\right) = \frac{b^2}{a^2}\left(\frac{\sec\theta + \tan\theta}{\sec\theta+1}\right) \ldots\ldots(7).$

$$\therefore\ a^2\tan^2\theta\sec\theta + a^2\tan^2\theta = b^2\sec\theta + b^2\tan\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8).$$

$$\therefore\ (a^2\tan^2\theta - b^2)\sec\theta = b^2\tan\theta - a^2\tan^2\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9).$$

Squaring (9) and substituting for $\sec^2\theta$ its value $1+\tan^2\theta$, performing operations indicated and arranging with reference to $\tan\theta$,

$$a^4\tan^6\theta - 2a^2b^2\tan^4\theta + 2a^2b^2\tan^3\theta - 2a^2b^2t^2 + b^4 = 0 \ldots\ldots\ldots(10).$$

Transposing the three middle terms and subtracting $2a^2b^2\tan^3\theta$,

$$a^4\tan^6\theta - 2a^2b^2\tan^3\theta + b^4 = 2a^2b^2(\tan^4\theta - 2\tan^3\theta + \tan^2\theta) \ldots\ldots(11).$$

Extracting square root, $a^2\tan^3\theta - b^2 = \pm ab(\tan^2\theta - \tan\theta)\sqrt{2} \ldots\ldots\ldots(12).$

$$\text{Whence } \tan\theta = \frac{\pm d\sqrt{2}}{3} + \frac{1}{3a}\left[\frac{b^2}{2}(9a \pm 4b\sqrt{2}) + \frac{3b}{2}(\pm 24a^3b\sqrt{2} - 39a^2b^2 \pm 24ab^3\sqrt{2})^{\frac{1}{2}}\right]^{\frac{1}{3}}$$

$$+ \frac{1}{3a}\left[\frac{b^2}{2}(9a \pm 4b\sqrt{2}) - \frac{3b}{2}(\pm 24a^3b\sqrt{2} - 39a^2b^2 \pm 24ab^3\sqrt{2})^{\frac{1}{2}}\right]^{\frac{1}{3}}.$$

From equation (5), $x = a\sqrt{\frac{1+\cos\theta}{2}} = a\cos\frac{1}{2}\theta$, and from equation (6)

$$y = b\sqrt{\frac{1+\sin\theta}{2}} = b\cos(\tfrac{1}{4}\pi - \tfrac{1}{2}\theta) = \frac{b}{\sqrt{2}}(\cos\tfrac{1}{2}\theta - \sin\tfrac{1}{2}\theta).$$

If $a=b$, from (12), $\tan\theta = 1$ or $\frac{\pm\sqrt{2}-1}{2} \pm \frac{1}{2}(\pm 2\sqrt{2}-1)^{\frac{1}{2}}$.

III. **Solution by J. SCHEFFER, A. M., Hagerstown, Maryland.**

Dividing (1) by (2) and putting $y = tx$, we obtain

$$x^2 = \frac{a^2b^2(1-t)}{2t(a^2t-b^2)}, \text{ and then } y^2 = \frac{a^2b^2t(1-t)}{2(a^2t-b^2)}.$$

Substituting these in (1), we obtain finally the equation

$$t^6-\frac{2b^2}{a^2}t^4+\frac{2b^2}{a^2}t^3-\frac{2b^2}{a^2}t^2+\frac{b^4}{a^4}=0.$$

Solving this for numerical values of a and b, we get the values of x and y from the above expressions.

The same equation may be arrived at by putting $x=r\cos\theta$, $y=r\sin\theta$. The given equation then changes into $a^2\cos\theta=2r^2\cos^2\theta-a^2$; $b^2\sin\theta=2r^2\sin^2\theta-b^2$. Adding, we get $a^2\cos\theta+b^2\sin\theta=2r^2-(a^2+b^2)$, whence $r^2=\frac{1}{2}[a^2\cos\theta+b^2\sin\theta+a^2+b^2]$. Also, $r^2=\frac{1}{2}.\dfrac{a^2\cos\theta+a^2}{\cos^2\theta}$.

Equalizing, changing into the tangent function, the latter being denoted by t, we obtain the same equation as above.

IV. Solution by H. C. WILKES, Skull Run, West Virginia.

Putting $x^2+y^2=s^2$; then from (1), $a^2(x+s)=2sx^2$, and from (2), $b^2(y+s)=2sy^2$. Any rational value for s will give integral [?] fractional values for a^2 and b^2. Let $s=5$, $a^2=45/4$, and $b^2=160/9$; $s=13$, $a^2=325/9$, and $b^2=3744/25$; $s=17$, $a^2=2176/25$, and $b^2=3825/16$.

67. Proposed by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tennessee.

Prove that $\cos\frac{n\pi}{7}+\cos\frac{3n\pi}{7}+\cos\frac{5n\pi}{7}=\frac{1}{2}$ or $-\frac{1}{2}$, according as n is *odd* or *even*, [and not a multiple of 7].

I. Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas-Texas.

$$\sin\frac{2n\pi}{7}=2\sin\frac{n\pi}{7}\cos\frac{n\pi}{7}.$$

$$\sin\frac{4n\pi}{7}-\sin\frac{2n\pi}{7}=2\sin\frac{n\pi}{7}\cos\frac{3n\pi}{7}.$$

$$\sin\frac{6n\pi}{7}-\sin\frac{4n\pi}{7}=2\sin\frac{n\pi}{7}\cos\frac{5n\pi}{7}.$$

$$\therefore\cos\frac{n\pi}{7}+\cos\frac{3n\pi}{7}+\cos\frac{5n\pi}{7}=\frac{\sin\frac{6}{7}n\pi}{2\sin\frac{1}{7}n\pi}=\frac{\sin(n\pi-\frac{1}{7}n\pi)}{2\sin\frac{1}{7}n\pi}$$

$=-\frac{1}{2}\cos n\pi=\pm\frac{1}{2}$, according as n is odd or even.

II. Solution by J. SCHEFFER, A. M., Hagerstown, Maryland.

Employing the well-known formula

$$\sum_{n=1}^{n=n}\cos[a+(n-1)b]=\frac{\cos[a+\frac{1}{2}(n-1)b]\sin\frac{1}{2}nb}{\sin\frac{1}{2}b},$$